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Performance Analysis of MIMO MRC Systems with Feedback Delay and Channel Estimation Error

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***Abstract*—In this paper, we investigate the performance of a multiple-input multiple-output (MIMO) maximal ratio combining (MRC) system with feedback delay and channel estimation error. By taking these practical imperfect factors into account, we first formulate the system model and derive the moment generating function (MGF) of the output signal-to-noise ratio (SNR) which serves as the basis for further system performance analysis. Then, we compute the probability density function (PDF) and the cumulative distribution function (CDF) of the output SNR. Further, we derive the analytical expressions of the exact and approximate average symbol error rates (SERs) of the MIMO MRC system, which are used to investigate the performance loss in terms of the array gain and diversity order. Finally, computer simulations are conducted to show the efficacy of the analytical results, and the effect of feedback delay and channel estimation error on the system performance.**

***Index Terms*—multiple-input multiple-output (MIMO), maximal ratio combining (MRC), feedback delay, channel estimation error, diversity order, performance analysis.**

I. INTRODUCTION

THE multiple-input multiple-output (MIMO) technique employing antenna array on both the transmitter and receiver sides has received considerable attention in the past decades, due to its ability of increasing the capacity and improving the link quality for wireless networks [1]-[3]. Among various MIMO strategies, MIMO maximal ratio combining (MRC), which applies the principle of beamforming (BF) to maximizing the output signal-to-noise ratio (SNR), is of particular interest, and has been widely studied in open literatures [4]-[17].

When perfect channel state information (CSI) is available at both of the transmitter and receiver, closed-form expressions of the probability density function (PDF) of the received SNR and average symbol error rate (SER) for the MIMO MRC systems have been obtained in [5] [6], and the outage probability and ergodic capacity have been derived in [7], where each entry of the channel matrix is assumed to be independent and identically distributed (i.i.d.). Performance analysis of double-correlated and arbitrarily correlated Rayleigh fading channels can be found in [8] and [9], respectively. Some statistical results of MIMO MRC systems over correlated Rician fading channels have been obtained in [10]. In practical MIMO systems, the receiver's combining vector is often calculated based on imperfect CSI due to the channel estimation error. Moreover, the transmit BF vector is chosen according to the outdated CSI because of the feedback delay. Under this situation, closed-form expressions for the moment generating function (MGF) and the PDF of the receive output SNR considering the effects of outdated and finite-rate feedback as well as further accurate analytical error rate expressions have been derived in [11]. In [12], the performance of transmit beamforming on multiple-antenna Rayleigh fading channels with imperfect channel feedback has been analyzed, where the feedback imperfections are characterized in terms of noisy channel estimation, feedback delay, and finite-rate channel quantization. The exact average SER, outage probability and ergodic capacity performance of beamforming in spatially correlated multiple-input single-output (MISO) systems with channel estimation error and feedback delay has been analyzed in [13]. However, the authors in [11]-[13] have limited their study to the MISO case, which can be regarded as a special case of MIMO MRC system with a single antenna at the receiver. More recently, many researchers have examined the effects of various errors on the MIMO MRC systems, where both the transmitter and receiver are equipped with multiple antennas. For example, the authors of [14] have investigated the effect of Gaussian estimation errors on the performance of maximal ratio transmission (MRT) in MIMO systems. In [15], the authors have studied the performance of MIMO wireless communications with transmit BF and receive MRC in the presence of channel estimation error and co-channel interferences. Furthermore, the joint impacts of channel estimation error, feedback delay and co-channel interference on the performance of MIMO systems employing MRT has been examined in [16]. It is worth-mentioning that the error term due to imperfect CSI is treated as noise or interference in [14]-[16]. This treatment is not highly efficient, since the error term can be demodulated via envelope detection in practical wireless systems, and thus can be used to enhance the output

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signal power [20]. Motivated by this observation, the authors of [17] have considered the error term as signal, and analyzed the bit error rate (BER) and outage probability (OP) of the singular value decomposition (SVD)-based MIMO systems with feedback delay and channel estimation error. However, the main drawback of the work in [17] is that only the channel gain of the signal term is regarded as random variable (RV). Actually, the channel gains of both the signal term and error term should be regarded as RVs in the performance analysis [11]. This observation motivates the work presented in this paper.

For reasons of clarity, we summarize the main contributions of this paper as follows:

•Based on the model of feedback delay and channel estimation error in [11], we present a closed-form expression of the output SNR for the MIMO MRC system, where a realistic scenario is considered by treating the error term due to imperfect CSI as signal, and assuming that the channel gains of both the signal and error terms are RVs.

•Analytical expressions are derived for the MGF, PDF and cumulative distribution function (CDF) of the output SNR to investigate the statistical properties of the MIMO MRC system with various errors, which is a general model containing the related work for example [11]-[13] as special cases. Meanwhile, these analytical results are different from those in [14]-[17] with accurate evaluation.

•New theoretical formulas are developed for the average symbol error rate (ASER) of the system, and a simple approximate ASER expression at high SNR is also derived to examine the asymptotic behavior of the MIMO MRC system conveniently.

The rest of the paper is organized as follows. In Section II, we formulate the model of the MIMO MRC system in the presence of the channel estimation error and feedback delay. In Section III, we give the derivation of the MGF, PDF and CDF for the output SNR. In Section IV, we work on closed-form expressions for the exact and approximate ASERs. Section V provides computer simulation results to validate our performance analysis, with respect to different errors, antenna configurations and modulation formats. Finally, Section VI draws the conclusion of our work in this paper.

Notations: Bold faced letters represent vectors or matrices, $(\cdot)^H$ the Hermitian transpose, $(\cdot)^*$ the complex conjugate, and $E[\,\cdot\,]$ the expectation. $\exp(\cdot)$ denotes the exponential function, $|\cdot|$ the absolute value, $\|\cdot\|_F$ the Frobenius norm, and $Q(\cdot)$ the Gaussian Q-function. χ_N^2 stands for a chi-square distributed random variable with N degrees of freedom, $\aleph_c(m,\sigma^2)$ for a complex Gaussian distribution with mean m and variance σ^2, and $\min\{a,b\}$ for the minimum of a and b. $diag\left(\begin{array}{cccc} a_1, & a_2, & \cdots, & a_N \end{array}\right)$ represents an $N\times N$ diagonal matrix with $\begin{array}{cccc} a_1, & a_2, & \cdots, & a_N \end{array}$ as its diagonal elements, $\mathbf{I}_N$ an $N\times N$ identity matrix, and $o(x^n)$ the terms with order higher than n.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a MIMO system with N_t antennas at the transmitter and N_r antennas at the receiver.

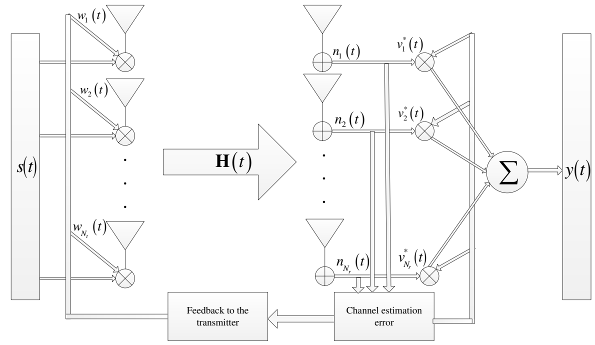

Fig. 1: System model of MIMO MRC with channel estimation error and feedback delay

The received signal can be written as

$$\mathbf{r}(t)=\mathbf{H}(t)\,\mathbf{w}(t)\,s(t)+\mathbf{n}(t)\,, \tag{1}$$

where $s(t)$ is the transmitted data symbol with transmit power $P_s = E\left[|s(t)|^2\right]$, $\mathbf{w}(t)$ is the $N_t\times 1$ transmit BF weight vector normalized to $\|\mathbf{w}(t)\|_F = 1$. In addition, $\mathbf{H}(t)$ represents the $N_r\times N_t$ Rayleigh fading channel matrix with its components $[\mathbf{H}(t)]_{i,j}$ being i.i.d. complex Gaussian random variables of zero mean and unit variance, i.e., $[\mathbf{H}(t)]_{i,j} \sim \aleph_c(0,1)$, and $\mathbf{n}(t)$ is an $N_r\times 1$ vector denoting the additive white Gaussian noise (AWGN) satisfying $\mathbf{n}(t)\sim\aleph_c(\mathbf{0},N_0\mathbf{I}_{N_r})$. After performing MRC with BF weight vector $\mathbf{v}(t)\,(N_r\times 1)$ satisfying $\|\mathbf{v}(t)\|_F = 1$, the output signal of the receiver is given by

$$y(t)=\mathbf{v}^H(t)\,\mathbf{r}(t)=\mathbf{v}^H(t)\,\mathbf{H}(t)\,\mathbf{w}(t)\,s(t)+\mathbf{v}^H(t)\,\mathbf{n}(t)\,, \tag{2}$$

Thus, the output SNR can be expressed as

$$\gamma(t)=\left|\mathbf{v}^H(t)\,\mathbf{H}(t)\,\mathbf{w}(t)\right|^2\gamma_s. \tag{3}$$

where $\gamma_s = E\left\{|s(t)|^2\right\}\Big/N_0 = P_s/N_0$ is the transmit SNR. In what follows, we aim to derive a closed-form expression of $\gamma(t)$ in terms of the available CSI.

Let us consider first the perfect CSI case. By applying the spectral theorem, the matrix $\mathbf{H}^H(t)\,\mathbf{H}(t)$ can be decomposed as

$$\begin{array}{l}\mathbf{H}^H(t)\,\mathbf{H}(t)=\left[\begin{array}{cccc}\mathbf{u}_1(t), & \mathbf{u}_2(t), & \cdots, & \mathbf{u}_{N_t}(t)\end{array}\right]\\ \times diag\left(\begin{array}{cccc}\lambda_1(t), & \lambda_2(t), & \cdots, & \lambda_{N_t}(t)\end{array}\right)\\ \times\left[\begin{array}{cccc}\mathbf{u}_1(t), & \mathbf{u}_2(t), & \cdots, & \mathbf{u}_{N_t}(t)\end{array}\right]^H,\end{array} \tag{4}$$

where $\lambda_i(t)'s$ are arranged in a non-increasing order. According to the principle of MIMO MRC [4], if perfect CSI is available at the transmitter, $\mathbf{w}(t)$ is chosen to be the eigenvector corresponding to the largest eigenvalue of $\mathbf{H}^H(t)\,\mathbf{H}(t)$, that is, $\mathbf{w}(t)=\mathbf{u}_1(t)$, and simultaneously, $\mathbf{v}(t)$ is given by $\mathbf{v}(t)=\mathbf{H}(t)\,\mathbf{u}_1(t)/\|\mathbf{H}(t)\,\mathbf{u}_1(t)\|_F$. Accordingly, the output SNR in (3) can be represented as

$$\gamma(t)=\lambda_1(t)\,\gamma_s. \tag{5}$$

In practice, since the CSI is estimated at the receiver and fedback to the transmitter through a delayed feedback channel as shown in Fig. 1, the transmit and receive BF vectors are calculated based on the delayed CSI with a channel estimation error [18] [19]. As such, the channel model with the estimation error and feedback delay can be described as

$$\mathbf{H}(t) = \rho_d \hat{\mathbf{H}}(t - T_d) + \mathbf{E}_e + \mathbf{E}_d, \tag{6}$$

where T_d is the feedback delay, $\hat{\mathbf{H}}(t)$ is the estimated channel matrix at the receiver, satisfying $\left[\hat{\mathbf{H}}(t)\right]_{i,j} \sim \aleph_c(0, 1 - \sigma_e^2)$, $\mathbf{E}_e = \mathbf{H}(t) - \hat{\mathbf{H}}(t)$ is the channel estimation error matrix with $[\mathbf{E}_e]_{i,j} \sim \aleph_c(0, \sigma_e^2)$. In (6), $\mathbf{E}_d = \hat{\mathbf{H}}(t) - \rho_d \hat{\mathbf{H}}(t - T_d)$ is the additional channel estimation error matrix caused by the feedback delay, obeying $[\mathbf{E}_d]_{i,j} \sim \aleph_c(0, \left(1 - |\rho_d|^2\right)(1 - \sigma_e^2))$, where ρ_d denotes the normalized correlation coefficient between $\left[\hat{\mathbf{H}}(t)\right]_{i,j}$ and its delayed sample $\left[\hat{\mathbf{H}}(t - T_d)\right]_{i,j}$, namely,

$$\rho_d = \frac{E\left\{\left[\hat{\mathbf{H}}(t)\right]_{i,j}\left[\hat{\mathbf{H}}(t - T_d)\right]_{i,j}^*\right\}}{1 - \sigma_e^2}, \tag{7}$$

Clearly, $\mathbf{E}_e$ is independent of $\hat{\mathbf{H}}(t)$ and $\mathbf{E}_d$. By letting $\hat{\mathbf{H}}(t - T_d) = \sqrt{1 - \sigma_e^2}\tilde{\mathbf{H}}(t - T_d)$ and the channel error matrix $\mathbf{E} = \mathbf{E}_e + \mathbf{E}_d$, the components of $\tilde{\mathbf{H}}(t - T_d)$ are i.i.d. with $\aleph_c(0, 1)$. Note that $\mathbf{E}$ is independent of $\tilde{\mathbf{H}}(t - T_d)$, whose entries are i.i.d. and obey $\aleph_c(0, 1 - |\rho_d|^2(1 - \sigma_e^2))$. The channel matrix can thus be rewritten as

$$\mathbf{H}(t) = \rho_d \sqrt{1 - \sigma_e^2}\tilde{\mathbf{H}}(t - T_d) + \mathbf{E}. \tag{8}$$

Since only the delayed version of the estimated CSI $\hat{\mathbf{H}}(t - T_d) = \sqrt{1 - \sigma_e^2}\tilde{\mathbf{H}}(t - T_d)$ is available at the transmitter, $\mathbf{w}(t)$ is calculated according to $\tilde{\mathbf{H}}(t - T_d)$. The eigendecomposition of $\tilde{\mathbf{H}}^H(t)\tilde{\mathbf{H}}(t)$ can be denoted as

$$\tilde{\mathbf{H}}^H(t)\tilde{\mathbf{H}}(t) = \tilde{\mathbf{U}}(t)\tilde{\mathbf{D}}(t)\tilde{\mathbf{U}}^H(t), \tag{9}$$

where $\tilde{\mathbf{D}}(t) = diag\left(\begin{array}{cccc}\tilde{\lambda}_1(t), & \tilde{\lambda}_2(t), & \cdots, & \tilde{\lambda}_{N_t}(t)\end{array}\right)$ with its diagonal elements arranged in a non-increasing order, and $\tilde{\mathbf{U}}(t) = \left[\begin{array}{cccc}\tilde{\mathbf{u}}_1(t) & \tilde{\mathbf{u}}_2(t) & \cdots & \tilde{\mathbf{u}}_{N_t}(t)\end{array}\right]$ with $\tilde{\mathbf{u}}_i(t)$ being the eigenvector corresponding to the eigenvalue $\tilde{\lambda}_i(t)$. As $\tilde{\mathbf{H}}^H(t - T_d)\tilde{\mathbf{H}}(t - T_d)$ is available at the transmitter, $\mathbf{w}(t)$ should be chosen as

$$\mathbf{w}(t) = \tilde{\mathbf{u}}_1(t - T_d), \tag{10}$$

Also, according to the principle of MRC at the receiver, $\mathbf{v}(t)$ is given by

$$\mathbf{v}(t) = \tilde{\mathbf{H}}(t - T_d)\tilde{\mathbf{u}}_1(t - T_d) \Big/ \left\|\tilde{\mathbf{H}}(t - T_d)\tilde{\mathbf{u}}_1(t - T_d)\right\|_F. \tag{11}$$

Furthermore, by considering the effect of various errors, and substituting (8), (10) and (11) into (2), the output signal of the receiver can be rewritten as

$$\begin{aligned} y(t) = & \underbrace{\rho_d\sqrt{1 - \sigma_e^2}\left\|\tilde{\mathbf{H}}(t - T_d)\tilde{\mathbf{u}}_1(t - T_d)\right\|_F s(t)}_{\text{signal term}} \\ & + \underbrace{\frac{\tilde{\mathbf{u}}_1^H(t - T_d)\tilde{\mathbf{H}}^H(t - T_d)}{\left\|\tilde{\mathbf{H}}(t - T_d)\tilde{\mathbf{u}}_1(t - T_d)\right\|_F}\mathbf{E}\tilde{\mathbf{u}}_1(t - T_d)s(t)}_{\text{error term}} \\ & + \underbrace{\frac{\tilde{\mathbf{u}}_1^H(t - T_d)\tilde{\mathbf{H}}^H(t - T_d)}{\left\|\tilde{\mathbf{H}}(t - T_d)\tilde{\mathbf{u}}_1(t - T_d)\right\|_F}\mathbf{n}(t)}_{\text{noise term}}, \end{aligned} \tag{12}$$

Compared with (2), it can be found that the error term due to the imperfect CSI is added in the output signal. Since the error term can be demodulated with envelope detection [20], it should be treated as signal as in [17] rather than the noise or interference as in [14]-[16]. As a result, after the necessary computations, the output SNR at the receiver can be expressed as

$$\begin{aligned} \gamma(t) = & \left|\rho_d\sqrt{1 - \sigma_e^2}\left\|\tilde{\mathbf{H}}(t - T_d)\tilde{\mathbf{u}}_1(t - T_d)\right\|_F \right. \\ & \left. + \frac{\tilde{\mathbf{u}}_1^H(t - T_d)\tilde{\mathbf{H}}^H(t - T_d)}{\left\|\tilde{\mathbf{H}}(t - T_d)\tilde{\mathbf{u}}_1(t - T_d)\right\|_F}\mathbf{E}\tilde{\mathbf{u}}_1(t - T_d)\right|^2 \gamma_s. \end{aligned} \tag{13}$$

Note that conditioned on $\tilde{\mathbf{H}}(t - T_d)$ and $\tilde{\mathbf{u}}_1(t - T_d)$, $\gamma(t)$ is a non-central chi-square distributed RV following $\gamma(t) \sim \chi_2^2$, whose sum of mean square and variance of each freedom are respectively, given by [20]

$$\tau = |\rho_d|^2\left(1 - \sigma_e^2\right)\tilde{\lambda}_1(t - T_d)\gamma_s, \tag{14}$$

$$\sigma^2 = \left[1 - |\rho_d|^2\left(1 - \sigma_e^2\right)\right]\gamma_s \Big/ 2. \tag{15}$$

Remark 1: When perfect CSI is available, we have $\rho_d = 1$, $\sigma_e^2 = 0$, $\tilde{\mathbf{H}}(t - T_d) = \mathbf{H}(t)$, $\tilde{\mathbf{u}}_1(t - T_d) = \mathbf{u}_1(t)$, and $\mathbf{E}$ reduces to an $N_r \times N_t$ null matrix. Thus, (13) is simplified to (5), which can also be found in some related works, such as [6, Eq. (9)] and [7, Eq. (5)], implying that the MIMO MRC system with perfect CSI considered in prior literatures is a special case of our general model.

Remark 2: According to the idea of [14, Eq. (6)], [15, Eq. (5)] and [16, Eq. (6)], the output SNR is given by

$$\gamma(t) = \frac{|\rho_d|^2\left(1 - \sigma_e^2\right)\tilde{\lambda}_1(t - T_d)\gamma_s}{\left[1 - |\rho_d|^2\left(1 - \sigma_e^2\right)\right]\gamma_s + 1}, \tag{16}$$

where the error term due to the imperfect CSI is treated as noise or interference. Meanwhile, following the work of [17, Eq. (11)], the output SNR can be expressed as

$$\gamma(t) = \left[|\rho_d|^2\left(1 - \sigma_e^2\right)\tilde{\lambda}_1(t - T_d) + 1 - |\rho_d|^2\left(1 - \sigma_e^2\right)\right]\gamma_s. \tag{17}$$

where only the channel gain of the signal term is regarded as RV. Clearly, compared with these related works investigating MIMO MRC system with feedback delay and channel estimation error, our work has a rigorous system model and clear physical meaning.

III. Output SNR Statistics

In this section, we study the statistical properties of the MIMO MRC system with feedback delay and channel estimation error, by deriving the closed-form expressions of MGF, PDF and CDF of the output SNR.

A. MGF of the output SNR

By using the conditional MGF $\psi_{\left(\gamma(t)|\tilde{\lambda}_1\left(t-T_d\right)\right)}(s)$ of $\gamma(t)$ on $\tilde{\lambda}_1(t-T_d)$ along with the PDF $f_{\tilde{\lambda}_1\left(t-T_d\right)}(x)$ of $\tilde{\lambda}_1(t-T_d)$, the MGF of $\gamma(t)$ in (13) can be expressed as

$$\psi_{\gamma(t)}(s) = \int_0^{\infty} \psi_{(\gamma(t)|x)}(s)\, f_{\tilde{\lambda}_1\left(t-T_d\right)}(x)\,dx, \tag{18}$$

Before deriving an analytical expression of (18), we first introduce the following theorem, which has been proven in [20]. It states that if X satisfies a non-central χ_N^2 with the variance of each freedom being σ^2 and the sum of mean square being τ, its PDF is given by

$$f_X(x) = \frac{1}{2\sigma^2}\left(\frac{x}{\tau}\right)^{\frac{N-2}{4}} \exp\left(-\frac{\tau+x}{2\sigma^2}\right) I_{\frac{N}{2}-1}\left(\sqrt{x}\frac{\tau}{\sigma^2}\right), \tag{19}$$

and its MGF can be further calculated as

$$\begin{aligned}&\psi_X(s)\\ &= \int_0^{\infty} f_X(x)\exp(sx)\,dx = \frac{1}{(1-2\sigma^2 s)^{N/2}} \exp\left(\frac{\tau s}{1-2\sigma^2 s}\right),\end{aligned} \tag{20}$$

Due to the fact that $\gamma(t) \sim \chi_2^2$, whose sum of mean square and variance of each freedom are given by (14) and (15), it is straightforward to obtain

$$\begin{aligned}&\psi_{\left(\gamma(t)|\tilde{\lambda}_1\left(t-T_d\right)\right)}(s) = \int_0^{\infty} f_{\left(\gamma(t)|\tilde{\lambda}_1\left(t-T_d\right)\right)}(x)\exp(sx)\,dx\\ &= \frac{1}{1-\left[1-\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\right]\gamma_s s} \exp\left\{\frac{\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\tilde{\lambda}_1\left(t-T_d\right)\gamma_s s}{1-\left[1-\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\right]\gamma_s s}\right\},\end{aligned} \tag{21}$$

Using (21) into (18) yields

$$\begin{aligned}&\psi_{\gamma(t)}(s)\\ &= \frac{1}{1-\left[1-\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\right]\gamma_s s}\\ &\times \int_0^{\infty} f_{\tilde{\lambda}_1\left(t-T_d\right)}(x)\exp\left\{\frac{\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\gamma_s s}{1-\left[1-\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\right]\gamma_s s}x\right\}dx\\ &= \frac{1}{1-\left[1-\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\right]\gamma_s s}\psi_{\tilde{\lambda}_1\left(t-T_d\right)}\left\{\frac{\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\gamma_s s}{1-\left[1-\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\right]\gamma_s s}\right\}\\ &= \frac{1}{1-\left[1-\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\right]\gamma_s s}\psi_{\tilde{\lambda}_1(t)}\left\{\frac{\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\gamma_s s}{1-\left[1-\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\right]\gamma_s s}\right\}.\end{aligned} \tag{22}$$

where $\psi_{\tilde{\lambda}_1\left(t-T_d\right)}(s)$ and $\psi_{\tilde{\lambda}_1(t)}(s)$ denote the MGF of $\tilde{\lambda}_1(t-T_d)$ and that of $\tilde{\lambda}_1(t)$, respectively. In (22), we have assumed that $\tilde{\lambda}_1(t-T_d)$ and $\tilde{\lambda}_1(t)$ have the same statistical property, due to the fact that the channel is ergodic.

Next, we consider the MGF of $\tilde{\lambda}_1(t)$. According to [6], the PDF of $\tilde{\lambda}_1(t)$ is given by

$$f_{\tilde{\lambda}_1(t)}(x) = \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m}\frac{i^{m+1}x^m\exp(-ix)}{m!}, \tag{23}$$

where $N_{tr} = \min\{N_t, N_r\}$, and the coefficient $d_{i,m}$ dependent on the transmit-receive antenna combinations can be expressed as [6]

$$d_{i,m} = \frac{\Gamma(i+1)\,C_{i,m}}{m^{i+1}\left(\prod_{l=1}^{N_{tr}}\Gamma(N_r-l+1)\,\Gamma(N_t-l+1)\right)}, \tag{24}$$

where $\Gamma(\cdot)$ is the Gamma function, and $C_{i,m}$ is the coefficient term of $x^i\exp(-mx)$. Note that X is a central χ_N^2 RV with the variance of each freedom being σ^2, its PDF and MGF are, respectively, given by [20]

$$f_X(x) = \frac{1}{2\sigma^2}\frac{1}{(N/2-1)!}x^{N/2-1}\exp\left(-\frac{x}{2\sigma^2}\right), \tag{25}$$

$$\psi_X(s) = \frac{1}{(1-2\sigma^2 s)^{N/2}}, \tag{26}$$

For the PDF of $\tilde{\lambda}_1(t)$ in (23), it can be viewed as a linear combination of PDFs with $N = 2(m+1)$ and $\sigma^2 = 1/2i$ given in (25), and weighted by $d_{i,m}$. Thus, the MGF of $\tilde{\lambda}_1(t)$ can be written as

$$\psi_{\tilde{\lambda}_1(t)}(s) = \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \frac{d_{i,m}}{(1-s/i)^{m+1}}, \tag{27}$$

Using (27) into (22), one can finally obtain the MGF of $\gamma(t)$ as

$$\psi_{\gamma(t)}(s) = \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m}\frac{(1-s/\alpha)^m}{(1-s/\beta)^{m+1}}. \tag{28}$$

where

$$\alpha = \frac{1}{\left[1-\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\right]\gamma_s}, \tag{29}$$

$$\beta = \frac{1}{\left[1-\left|\rho_d\right|^2\left(1-\sigma_e^2\right)\left(1-\frac{1}{i}\right)\right]\gamma_s}. \tag{30}$$

Remark 3: If only a single antenna is employed at the receiver, and simultaneously, the channel estimation error is ignored, we have $N_{tr} = N_r = 1$, $\sigma_e^2 = 0$, and $d_{1,N_t-1} = 1$. Thus $\psi_{\gamma(t)}(s) = \frac{(1-s/\alpha)^{N_t-1}}{(1-s/\beta)^{N_t}}$ with $\alpha = \frac{1}{\left(1-\left|\rho_d\right|^2\right)\gamma_s}$ and $\beta = \frac{1}{\gamma_s}$, which coincides with [11, Eq. (9)], indicating that the work in [11] is a special case of our result.

B. PDF of the output SNR

By applying the inverse Laplace transform and the MGF expression of $\gamma(t)$ in (28), the PDF of the output SNR can be obtained as

$$\begin{aligned}&f_{\gamma(t)}(x) = L^{-1}\left[\psi_{\gamma(t)}(s)\right]\\ &= \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m}L^{-1}\left[\frac{(1-s/\alpha)^m}{(1-s/\beta)^{m+1}}\right]\\ &= \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \sum_{n=0}^{m} d_{i,m}\frac{C_m^n}{(m-n)!}\frac{\beta^{m+1}}{\alpha^n}\\ &\times(1-\beta/\alpha)^{m-n}x^{m-n}\exp(-\beta x),\end{aligned} \tag{31}$$

where $L^{-1}[\cdot]$ denotes the inverse Laplace transform, which can be calculated with the help of the residue theorem as in [11].

Remark 4: For the MISO case as discussed in [11], we have $N_{tr} = N_r = 1$, $\sigma_e^2 = 0$, and $d_{1,N_t-1} = 1$, and thus (31) is simplified to

$$\begin{aligned} f_{\gamma(t)}(x) = & \sum_{n=0}^{m} \frac{C_{N_t-1}^n}{(N_t-n-1)!} \frac{\beta^{N_t}}{\alpha^n} (1-\beta/\alpha)^{N_t-n-1} \\ & \times x^{N_t-n-1} \exp(-\beta x). \end{aligned} \tag{32}$$

which is the same as [11, Eq. (13)].

C. CDF of the output SNR

The CDF of $\gamma(t)$ can be calculated from the integral of (31), that is,

$$\begin{aligned} F_{\gamma(t)}(x) = & \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \sum_{n=0}^{m} d_{i,m} \frac{C_m^n}{(m-n)!} \\ & \times \frac{\beta^{m+1}}{\alpha^n} (1-\beta/\alpha)^{m-n} \int_0^x t^{m-n} \exp(-\beta t)\, dt, \end{aligned} \tag{33}$$

By applying the identity [21]

$$\int_0^x t^m \exp(-\alpha t)\, dt = \frac{m!}{\alpha^{m+1}} \left[1 - \exp(-\alpha x) \sum_{k=0}^{m} \frac{(\alpha x)^k}{k!}\right], \tag{34}$$

the CDF of the output SNR in (33) can be calculated as

$$\begin{aligned} F_{\gamma(t)}(x) = & \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \sum_{n=0}^{m} C_m^n \left(\frac{\beta}{\alpha}\right)^n \left(1-\frac{\beta}{\alpha}\right)^{m-n} \\ & \times \left[1 - \exp(-\beta x) \sum_{k=0}^{m-n} \frac{(\beta x)^k}{k!}\right] \\ = & \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} - \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \sum_{n=0}^{m} \sum_{k=0}^{m-n} d_{i,m} \\ & \times C_m^n \left(\frac{\beta}{\alpha}\right)^n \left(1-\frac{\beta}{\alpha}\right)^{m-n} \frac{(\beta x)^k}{k!} \exp(-\beta x), \end{aligned} \tag{35}$$

With the help of the following equation [22]

$$\sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} = 1, \tag{36}$$

the CDF of $\gamma(t)$ can be further written in a simple closed-from expression as

$$\begin{aligned} F_{\gamma(t)}(x) = 1 - & \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \sum_{n=0}^{m} \sum_{k=0}^{m-n} d_{i,m} \\ & \times C_m^n \left(\frac{\beta}{\alpha}\right)^n \left(1-\frac{\beta}{\alpha}\right)^{m-n} \frac{(\beta x)^k}{k!} \exp(-\beta x). \end{aligned} \tag{37}$$

Remark 5: It is noted that the expression of (37) is reduced to

$$\begin{aligned} F_{\gamma(t)}(x) = 1 - & \sum_{n=0}^{N_t-1} \sum_{k=0}^{N_t-n-1} C_{N_t-1}^n \left(\frac{\beta}{\alpha}\right)^n \\ & \times \left(1-\frac{\beta}{\alpha}\right)^{N_t-n-1} \frac{(\beta x)^k}{k!} \exp(-\beta x). \end{aligned} \tag{38}$$

with $N_{tr} = N_r = 1$, $\sigma_e^2 = 0$, and $d_{1,N_t-1} = 1$ for the MISO scenario without channel estimation error. Interestingly, (38) coincides with [23, Eq. (26)], meaning that we have indeed extended the prior CDF results on MISO systems to the MIMO MRC case.

IV. Average SER

In this section, we analyze the average SER (ASER) of the MIMO MRC system with feedback delay and channel estimation error. In particular, we derive closed-form expressions of the exact ASER with various modulation formats. Furthermore, we develop a novel approximate ASER at high SNR to investigate the array gain and diversity order of the system in the presence of various errors.

A. Exact ASER

It is well-known that the SER of a wireless system with various modulation formats over AWGN channel is given by [20]

$$P_s(\gamma(t)|\, a, b) = aQ\left(\sqrt{2b\gamma(t)}\right), \tag{39}$$

where a and b are the parameters specified by the modulations. For example, for *M*-ary pulse amplitude modulation (*M*-PAM), we have $a = 2(M-1)/M$ and $b = 3/(M^2-1)$; a good approximation for *M*-ary phase shift keying (*M*-PSK) is given by $a = 2$ and $b = \sin^2(\pi/M)$; and a tight upper bound for *M*-ary quadrature amplitude modulation (*M*-QAM) is specified by $a = 4$ and $b = 3/2(M-3)$. Furthermore, the ASER over fading channels can be written as

$$\begin{aligned} P_s(a,b) & = E_{\gamma(t)}\{P_s(\gamma(t)|\, a,b)\} \\ & = \int_0^\infty aQ\left(\sqrt{2bz}\right) f_{\gamma(t)}(z)\, dz, \end{aligned} \tag{40}$$

Substituting (31) into (40) yields

$$\begin{aligned} P_s(a,b) = a & \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \sum_{n=0}^{m} d_{i,m} \frac{C_m^n}{(m-n)!} \frac{\beta^{m+1}}{\alpha^n} \\ & \times (1-\beta/\alpha)^{m-n} \int_0^\infty z^{m-n} \exp(-\beta z)\, Q\left(\sqrt{2bz}\right) dz, \end{aligned} \tag{41}$$

By utilizing the following equality [21]

$$\begin{aligned} & \frac{a^m}{\Gamma(m)} \int_0^\infty \exp(-at)\, t^{m-1} Q\left(\sqrt{bt}\right) dt \\ & = \frac{1}{2}\left[1 - \mu \sum_{k=0}^{m-1} C_{2k}^k \left(\frac{1-\mu^2}{4}\right)^k\right], \end{aligned} \tag{42}$$

where $\mu = \sqrt{\frac{b}{2a+b}}$, the ASER is thus given by

$$\begin{aligned} P_s(a,b) = \frac{a}{2} & \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \sum_{n=0}^{m} d_{i,m} C_m^n \left(\frac{\beta}{\alpha}\right)^n \\ & \times \left(1-\frac{\beta}{\alpha}\right)^{m-n} \left[1 - \mu \sum_{k=0}^{m-n} C_{2k}^k \left(\frac{1-\mu^2}{4}\right)^k\right]. \end{aligned} \tag{43}$$

where $\mu = \sqrt{\frac{b}{\beta+b}}$. Therefore, (43) generalizes the ASER expression of [6] with perfect CSI to the case of considering the channel estimation error and feedback delay.

B. Approximate ASER at high SNR

According to [22], an alternative calculation of the ASER can be expressed as

$$P_s(a,b) = \frac{a}{2}\sqrt{\frac{b}{\pi}} \int_0^\infty \frac{\exp(-bu)}{\sqrt{u}} F_{\gamma(t)}(u)\, du, \tag{44}$$

In order to obtain an accurate approximation for (44), we first present the following theorem. (see Appendix A for proof)

Theorem 1: The first-order expansion of the CDF of $\gamma(t)$ given by (37) can be expressed as

$$\begin{aligned} F_{\gamma(t)}(x) = & \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{\beta^{m+1}}{\alpha^m} x \\ & + \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{\beta^m}{\alpha^m} o(\beta x), \end{aligned} \tag{45}$$

Substituting (45) into (44), we have

$$\begin{aligned} & P_s(a,b) \\ & = \underbrace{\frac{a}{2}\sqrt{\frac{b}{\pi}} \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{\beta^{m+1}}{\alpha^m} \int_0^\infty \exp(-bu)\sqrt{u}du}_{I_1} \\ & + \underbrace{\frac{a}{2}\sqrt{\frac{b}{\pi}} \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{\beta^m}{\alpha^m} \int_0^\infty \frac{\exp(-bu)}{\sqrt{u}} o(\beta u)\, du}_{I_2}, \end{aligned} \tag{46}$$

I_1 can be calculated with the integral formula [24, Eq. (3.381.4)] as

$$\begin{aligned} I_1 = & \tfrac{a}{4b} \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \\ & \times \frac{\left[1-|\rho_d|^2\left(1-\sigma_e^2\right)\right]^m}{\left[1-|\rho_d|^2(1-\sigma_e^2)\left(1-\frac{1}{i}\right)\right]^{m+1}} \frac{1}{\gamma_s}, \end{aligned} \tag{47}$$

Meanwhile, the integral in I_2 can be denoted with the help of [24, Eq. (3.381.4)] as

$$\begin{aligned} & \int_0^\infty \tfrac{\exp(-bu)}{\sqrt{u}} o(\beta u)\, du \\ & = \sum_{i=2}^\infty k_i \beta^i \int_0^\infty u^{i-1/2} \exp(-bu)\, du \\ & = \sum_{i=2}^\infty k_i \tfrac{\Gamma(i+1/2)}{b^{i+1/2}} \beta^i \\ & = o(\beta), \end{aligned} \tag{48}$$

From (29), (30) and (48), and after some mathematical manipulations, I_2 in (46) can be derived as

$$\begin{aligned} I_2 = & \tfrac{a}{2}\sqrt{\tfrac{b}{\pi}} \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{\left[1-|\rho_d|^2\left(1-\sigma_e^2\right)\right]^m}{\left[1-|\rho_d|^2(1-\sigma_e^2)\left(1-\frac{1}{i}\right)\right]^m} \\ & \times o\left(\frac{1}{1-|\rho_d|^2(1-\sigma_e^2)\left(1-\frac{1}{i}\right)} \frac{1}{\gamma_s}\right), \end{aligned} \tag{49}$$

Noting that (49) can be regarded as a linear combination of $o\left(\frac{1}{\gamma_s}\right)$, I_2 can be written as $I_2 = o\left(\frac{1}{\gamma_s}\right)$. Therefore, with the expressions of I_1 and I_2, the ASER can be expressed as

$$P_s(a,b) = (G_a \gamma_s)^{-G_d} + o\left(\gamma_s^{-G_d}\right). \tag{50}$$

where G_a and G_b represent the array gain and the diversity order of the MIMO MRC system with various errors, which

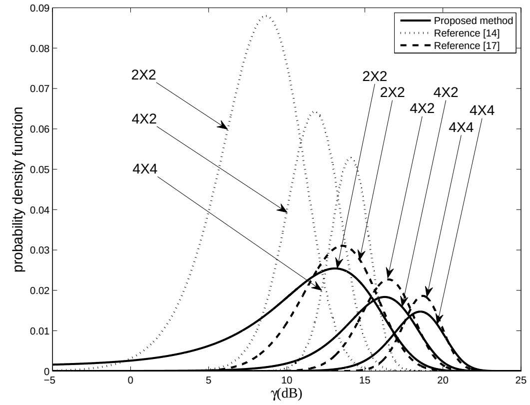

Fig. 2: PDF of the MIMO MRC system with different antenna combinations and various SNR expressions

are, respectively, given by

$$G_a = \frac{4b}{a \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{\left[1-|\rho_d|^2(1-\sigma_e^2)\right]^m}{\left[1-|\rho_d|^2(1-\sigma_e^2)\left(1-\frac{1}{i}\right)\right]^{m+1}}}, \tag{51}$$

$$G_d = 1, \tag{52}$$

It is to be noted that when SNR γ_s is very large, we have $o\left(\gamma_s^{-G_d}\right) \approx 0$. Thus, the ASER at high SNR can be approximated as

$$P_s(a,b) = (G_a \gamma_s)^{-G_d}. \tag{53}$$

It has already been proven in [25] that the diversity order of MIMO MRC system equals $N_t N_r$, when perfect CSI is available at both sides. However, from (52), we can find that the diversity order reduces to one due to the existence of channel estimation error and feedback delay, no matter how many antennas are employed. This scenario implies that the quality of CSI significantly affects the performance of the MIMO MRC system. The same conclusion has also been drawn in [11], where the diversity order of the transmit beamforming in MISO system impaired by feedback delay is also reduced to one while the diversity order of the transmit BF with perfect CSI is N_t.

V. Computer Simulation

In this section, we carry out computer simulations to validate the theoretical results obtained in previous sections, and to investigate the system performance in various scenarios with different channel estimation errors, antenna configurations and modulation formats. In our simulation, we apply the Jake's fading to model the error, namely, $\rho_d = J_0(2\pi f_m \tau)$, with $f_m \tau$ being the Doppler spread rate ratio and $J_0(\cdot)$ the zero order Bessel function of the first kind, and set the transmit SNR as $\gamma_s = 10dB$ in Fig.2$\sim$Fig.7. In all plots, the label $N_t \times N_r$ means the combination of antennas at the transmitter and the receiver.

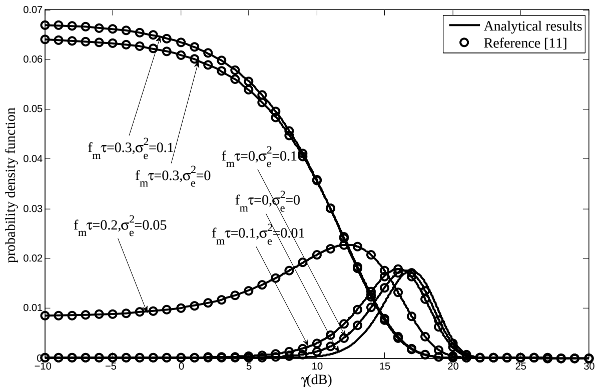

Fig. 3: PDF of the 6×1 MIMO MRC system with various parameters

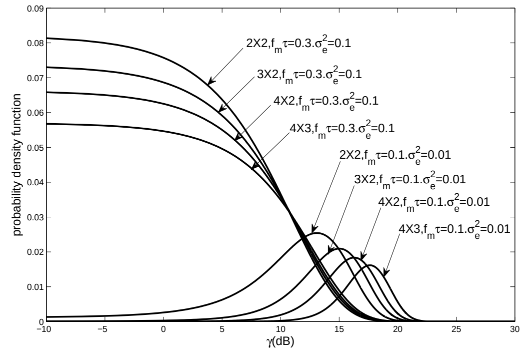

Fig. 5: PDF of the MIMO MRC system with different antenna combinations and various parameters

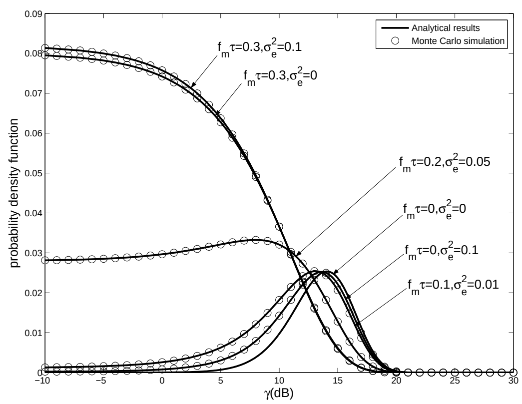

Fig. 4: PDF of the 2×2 MIMO MRC system with various parameters

First of all, we investigate the PDF of the proposed method by comparing with the previous works such as [14] and [17]. Supposing that the values of feedback delay and channel estimation error are chosen as $f_m\tau = 0.1$ and $\sigma_e^2 = 0.01$, respectively, we consider the following three antenna combinations: 2×2, 4×2 and 4×4, and plot the PDF curves of the output SNR in Fig.2. Here, the results of [17] are obtained without taking the co-channel interference into account. As we expect, the PDF curves obtained from our method and [17] shift to higher SNR values than those from [14], which attributes to the different treatment of the error term due to imperfect CSI. Meanwhile, it can also be observed that due to the different treatment of the channel gain owing to the error term, the PDF peaks of our method are different from those of [17]. Furthermore, we find that a better performance can be obtained as the number of antennas increases, which clearly demonstrates the advantage of employing multiple antennas.

Secondly, we study the impact of various errors on the PDF of the MIMO MRC system. The validity of the theoretical results of our paper is verified by the comparison between eq. (31) and eq. (15) in [11]. The PDF simulation results are given in Fig.3 and the antenna configuration is assumed to be 6×1. As we can see that the theoretical formulas in [11] match our analytical results perfectly for different values of $f_m\tau$ and σ_e^2, which demonstrate that the PDF formula of [11] is a special case of our general result. Fig. 4 depicts the theoretical and simulated PDFs of a 2×2 system with various feedback delays and different channel estimation errors. It is seen that the PDF curves from the Monte Carlo experiments are highly consistent with those given by theoretical formulas regardless of different values of $f_m\tau$ and σ_e^2. This implies that the derived exact expression in (31) can accurately evaluate the PDF of the output SNR. As for different parameters, four different cases are considered in our simulation, that is, (i) MIMO MRC system without channel estimation error and feedback delay, namely, $f_m\tau = 0, \sigma_e^2 = 0$; (ii) only with feedback delay, namely, $f_m\tau = 0.3, \sigma_e^2 = 0$; (iii) only with channel estimation error, namely, $f_m\tau = 0, \sigma_e^2 = 0.1$; and (iv) with both feedback delay and channel estimation error. Here, case (iv) is further divided into three scenarios: short feedback delay and small variance of channel estimation error, namely, $f_m\tau = 0.1, \sigma_e^2 = 0.01$; moderate feedback delay and variance of channel estimation error, namely, $f_m\tau = 0.2, \sigma_e^2 = 0.05$; and long feedback delay and large variance of channel estimation error, namely, $f_m\tau = 0.3, \sigma_e^2 = 0.1$. It is found that the perfect CSI case corresponds to the best performance for its curve shifting towards the higher value of SNR. For fixed feedback delay, the performance becomes worse with the increase of channel estimation error. For fixed channel estimation error, the performance gets better with shorter feedback delay. In addition, Fig. 5 gives the PDF of the MIMO MRC system with different antenna combinations and various parameters. For the cases with the same parameters, the performance becomes better as the number of transmit antennas increases, such as from 2×2 to 3×2, or the number of receive antennas increases, e.g. 4×2 vs 4×3. This is because more array gain is provided by increasing the antenna number. For the same antenna combination, it is obvious that the cases with short feedback delay and small variance of channel estimation error have better performance.

Thirdly, we present the CDF of the output SNR in Fig. 6 for a 3×2 antenna combination and in Fig. 7 for different antenna combinations, both with various parameters. We can see the theoretical curves match very well with the Monte

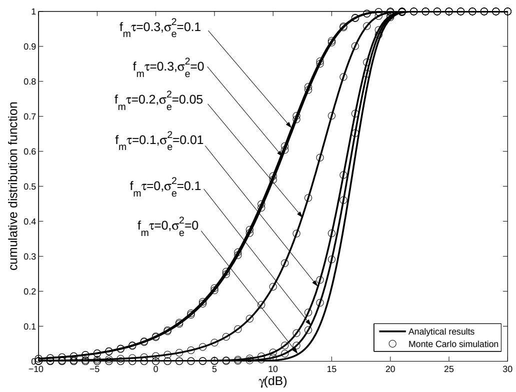

Fig. 6: CDF of the 3×2 MIMO MRC system with various parameters

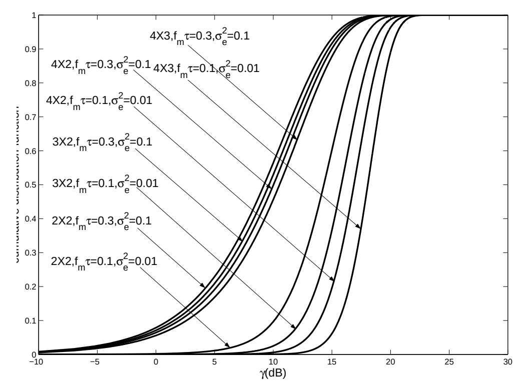

Fig. 7: CDF of the MIMO MRC system with different antenna combinations and various parameters

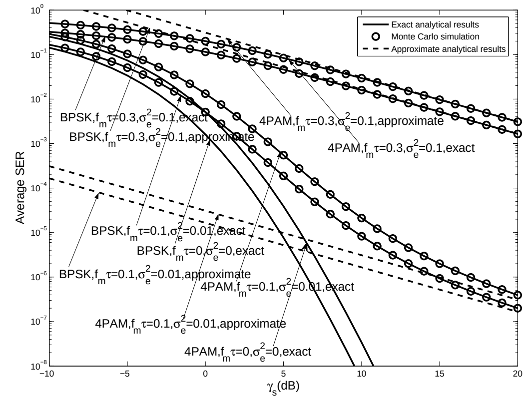

Fig. 8: Average SER of the 4×2 MIMO MRC system with various parameters and modulations

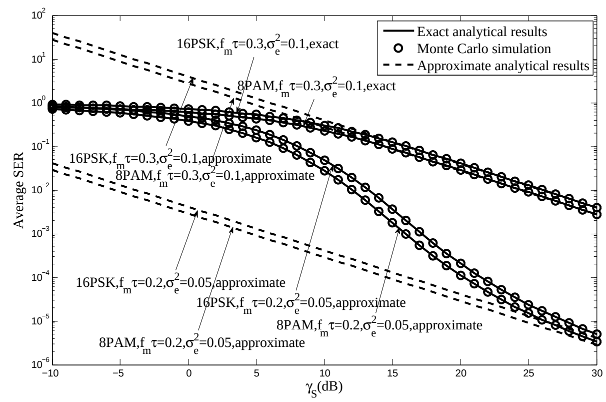

Fig. 9: Average SER of the 8×4 MIMO MRC system with various parameters and modulations

Carlo simulation results. It is known that for the same SNR, the performance upgrades with the smaller CDF value. Thus, a similar conclusion as in Fig. 4 can be drawn, namely, a better performance can be obtained with shorter feedback delay and smaller variance of channel estimation error. In Fig. 7, it is shown that a better performance is achieved by setting short feedback delay and small variance of channel estimation error for the same antenna combination which coincides with the result in Fig. 5.

Finally, we examine the validity of the derived exact ASER expression and its approximation at high SNR. In Fig. 8, we plot the average SER with various parameters and modulation formats for a 4×2 antenna combination. Two modulations, BPSK and 4PAM are included in our simulation. Three cases, $f_m\tau = 0, \sigma_e^2 = 0$, $f_m\tau = 0.1, \sigma_e^2 = 0.01$ and $f_m\tau = 0.3, \sigma_e^2 = 0.1$ are considered. We can see a good match between the Monte Carlo simulations and the exact analytical results. As well, the approximate analytical results approach closely the exact results at high SNR for various parameters and modulation formats. Compared with the perfect CSI case, the case with feedback delay and channel estimation error has a large performance loss as the diversity order has been reduced from N_tN_r to 1. It is obvious that the ASER of BPSK is better than that of 4PAM for the same parameters. Also, the case with $f_m\tau = 0.1, \sigma_e^2 = 0.01$ has a better ASER than the one with $f_m\tau = 0.3, \sigma_e^2 = 0.1$ due to a shorter feedback delay and smaller variance of channel estimation error. The average SER with various parameters and modulation formats for a 8×4 antenna combination is also given in Fig. 9, where 8PAM and 16PSK are investigated for $f_m\tau = 0.2, \sigma_e^2 = 0.05$ and $f_m\tau = 0.3, \sigma_e^2 = 0.1$. A good match between the Monte Carlo simulations and the exact analytical results as well as the approximate analytical results approaching closely the exact results at high SNR show the validity of our ASER formula and the asymptotic ASER results. In Fig. 10, we present the exact ASER and the approximate ASER at high SNR for different antenna combinations, where the parameters are set to be $f_m\tau = 0.2, \sigma_e^2 = 0.05$, and the modulation format is BPSK. The approximate ASER approaches very closely the exact ASER at high SNRs for various antenna combinations, showing a good robustness of our analytical results. Furthermore, although the antenna combinations with more transmit or receive antennas give a better performance, they have the same diversity order as shown by the respective curves which have the same slope at high SNRs.

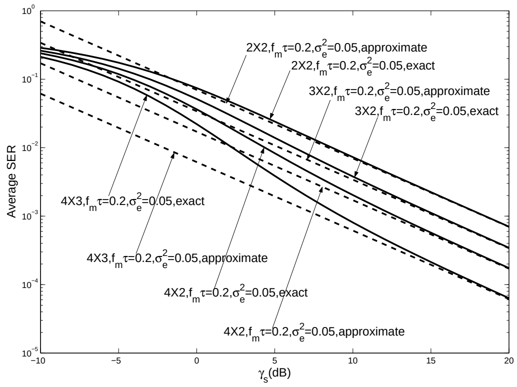

Fig. 10: Average SER of the MIMO MRC system with different antenna combinations and various parameters in the case of BPSK modulation

VI. Conclusion

In this paper, we have analyzed the MIMO MRC systems with feedback delay and channel estimation error. Closed-form expressions for the PDF and CDF of the output SNR have been obtained. The exact and approximate average SERs of the system have also been derived. It is shown that the channel estimation error and the feedback delay have reduced the diversity order from $N_t N_r$ to 1 and thus caused a significant performance loss. Computer simulations have confirmed the validity of the performance analysis and shown the performance difference among the systems with different channel estimation errors, feedback delays, antenna configurations and modulation formats.

Appendix A
Proof of Theorem 1

The first-order expansion of $1-\exp\left(-\eta x\right)\sum\limits_{k=0}^{m}\frac{(\eta x)^k}{k!}$ can be expressed as

$$1-\exp\left(-\eta x\right)\sum_{k=0}^{m}\frac{(\eta x)^k}{k!}=\frac{(\eta x)^{m+1}}{(m+1)!}+o\left((\eta x)^{m+1}\right), \quad (54)$$

Substituting (54) into the first line of (35), we have

$$\begin{aligned}F_{\gamma(t)}\left(x\right)=&\sum_{i=1}^{N_{tr}}\sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2}d_{i,m}\sum_{n=0}^{m}C_m^n\left(\tfrac{\beta}{\alpha}\right)^n\\&\times\left(1-\tfrac{\beta}{\alpha}\right)^{m-n}\left[\tfrac{(\beta x)^{m-n+1}}{(m-n+1)!}+o\left((\beta x)^{m-n+1}\right)\right],\end{aligned} \quad (55)$$

In order to derive the first-order expansion of $F_{\gamma(t)}\left(x\right)$, we need to find the first nonzero coefficient in the Maclaurin expansion. Letting $n=m$, one can obtain the first-order expansion of $\sum\limits_{n=0}^{m}C_m^n\left(\frac{\beta}{\alpha}\right)^n\left(1-\frac{\beta}{\alpha}\right)^{m-n}\left[\frac{(\beta x)^{m-n+1}}{(m-n+1)!}+o\left((\beta x)^{m-n+1}\right)\right]$ as $\frac{\beta^{m+1}}{\alpha^m}x+\frac{\beta^m}{\alpha^m}o\left(\beta x\right)$, and hence the first-order expansion of the CDF is finally given by

$$\begin{aligned}F_{\gamma(t)}\left(x\right)=&\sum_{i=1}^{N_{tr}}\sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2}d_{i,m}\frac{\beta^{m+1}}{\alpha^m}x\\&+\sum_{i=1}^{N_{tr}}\sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2}d_{i,m}\frac{\beta^m}{\alpha^m}o\left(\beta x\right).\end{aligned} \quad (56)$$

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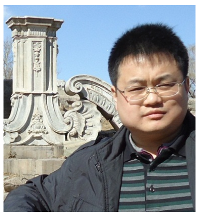

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